

# Chern-Simons Supersymmetric Branes

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## Abstract

The purpose of this Letter is to continue the study of the class of models proposed in the previous paper [1] hep-th/0002077. The model corresponds to a system of branes of diverse dimensionalities with Chern-Simons actions for a supergroup, embedded in a background described also by a Chern-Simons action. The model treats the background and the branes on an equal footing, providing a 'brane-target space democracy'. Here we suggest some possible extensions of the original model, and discuss its equations of motion, as well as the issue of currents and charges carried by the branes. We also discuss the relationship with M-theory and Superstring theory.

## 1. Introduction

Chern-Simons Supergravity (CSS) theories in diverse dimensions have been studied in many papers during the last years [9, 10, 11, 13, 12, 14, 17]. Those theories are very interesting on their own right, having a wealth of nice properties, ranging from being topological (in the sense of being independent of any background metric), being true gauge theories for extensions of the standard space-time symmetry groups (like the Poincaré or anti de Sitter group), having coupling constants that are not renormalized (so that the classical action is also the quantum effective action) and being exactly solvable in 2+1 dimensions. Some time ago Chamseddine [10] suggested that this kind of models might be regarded as the basis for an approach to the unification of the fundamental interactions alternative to the Superstring Theory [18] program. More recently Horava [17] proposed that a CSS may correspond to the M-theory [22, 21, 20, 19], the fundamental theory underlying the five known consistent superstring theories, which would then be an ordinary field theory (against the belief of most experts on the field).

On other line of development it was attempted to recast the 1+1 dimensional superstring actions as 2+1 CS theories [15, 16] (see also [9]) by a sort of 'thickening of the world sheet', as a way to benefit from the good properties of the later.

Recently we suggested [1] a way to introduce fundamental supersymmetric extended objects on CSS, inspired on the model of ref.[23] for the coupling of branes to Yang-Mills fields, which brought together these approaches. There is a model of that class which has at least two of the superstring theories (IIA and IIB) as sectors of its phase space and it describe branes with actions of the same form that the one describing the background, which is a CSS with the branes acting as sources for the (super)gauge fields, and interacting with that background. Also it is plausible that standard supergravity approximately describes some regime of the theory [13, 17, 14]. It was then argued in [1] that a model of the kind considered there might correspond to the quantum effective action of the M-theory.

In this paper I continue the study of that kind of models by reviewing its invariances and transformation properties, discussing its equations of motion and the issue of currents and charges carried by the branes. Finally I discuss several of the questions and open problems where I believe further developments are to be expected. The model of this paper differs from the one on ref.[1] in the fact that we point out that it may not be necessary to introduce a kinetic term 'by hand' in order to make contact with superstring theory.

## 2. The Action

The mathematical tools used through this paper are discussed in (and were to a great extent developed in) references.[2, 3, 4, 5]. We will follow the notation and conventions of ref.[5]<sup>1</sup>. We consider a (super)group  $G$  with generators  $T^I$ , the gauge potential 1-form  $A = A_m^I T^I dx^m$  and the curvature 2-form  $F = dA + A^2$ , defined on a manifold  $M^{2N+2}$  of even dimension  $2N$ . The covariant derivative is  $D = d + [A, \cdot]$  and it follows that  $F$  satisfy the Bianchi identity  $DF = 0$ . An invariant polynomial  $P(F)$  is defined as the formal sum

$$P(F) = \sum_{n=0}^N \alpha_n \text{STr} (F^{n+1}) \quad , \quad (1)$$

where  $\text{STr} (T^{I_1} \dots T^{I_{n+1}}) = g^{I_1 \dots I_{n+1}}$  stands for an invariant symmetric supertrace on the algebra of  $G$ . An important property is

$$d \text{STr} ( \quad ) = \text{STr} (D \quad ) \quad (2)$$

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<sup>1</sup>An excellent recent work covering the mathematical background of these articles is the book on anomalies by R. Bertlmann [6]. The reader may find amusing to know that Dr. Bertlmann socks have inspired in J.S. Bell some deep reflections on the foundations of quantum mechanics [7]. Bertlmann reminiscences of J.S. Bell on the topic of anomalies (where he was one of the pioneers) can be found in the Preface to [6].

We define the action for our system by

$$S = \sum_{n=0}^N \alpha_n \int_{S^{2n+1}} k_{01} \text{STr} (F^{n+1}) \quad (3)$$

where  $S^{2N+1}$  is the boundary of  $M^{2N+2}$ ,  $S^{2N+1} \equiv \partial M^{2N+2}$ , and the manifolds  $S^{2n+1}$  are embedded into  $S^{2N+1}$ . In case  $S^{2N+1}$  itself has a boundary  $\Omega^{2N}$ , then  $S^{2N+1}$  is included into the boundary of  $M^{2N+2}$ . The manifolds  $S^{2n+1}$  may have boundaries in manifolds  $\Omega^{2n}$ .

The Cartan homotopy operator acting on polynomials  $\mathcal{P}(F_t, A_t)$ , with  $A_t$  interpolating between two gauge potentials  $A_0$  and  $A_1$  as

$$A_t = tA_1 + (1-t)A_0 \quad , \quad F_t = dA_t + A_t^2 \quad . \quad (4)$$

is defined to be

$$k_{01}\mathcal{P}(F_t, A_t) = \int_0^1 dt \, l_t \mathcal{P}(F_t, A_t) \quad , \quad (5)$$

with the operator  $l_t$  defined to act on arbitrary polynomials by

$$l_t A_t = 0 \quad , \quad l_t F_t = A_1 - A_0 \equiv J \quad , \quad (6)$$

and the convention that  $l_t$  is defined to act as an antiderivation.

The dynamical variables are the gauge potentials  $A_m^I$ , the embedding coordinates of the submanifolds  $S^{2n+1}$ ,  $X_{(2n+1)}^m(\chi_{(2n+1)}^i)$ ,  $m = 0, \dots, 2N+1$  where the  $\chi_{(2n+1)}^i$  with  $i = 0, \dots, 2n+1$ , are local coordinates in  $S^{2n+1}$  and the embedding coordinates of the submanifolds  $\Omega^{2n}$ ,  $X_{(2n)}^m(\xi_{(2n+1)}^i)$ ,  $m = 0, \dots, 2N+1$  where the  $\xi_{(2n)}^i$  with  $i = 0, \dots, 2n$ , are local coordinates in  $\Omega^{2n}$  (of course in the boundary  $\Omega^{2n}$  of  $S^{2n+1}$  the  $X^m$ 's of the same point must coincide as functions of the  $\chi$ 's or the corresponding  $\xi$ 's). Notice that all these manifolds are supposed to be non compact at least in what would be the 'time direction'. It was noticed in Refs.[12] that the dimensionless coefficients  $\alpha_n$  can consistently take only a discrete set of values if we require that the quantum theory must be independent of the way in which the manifolds  $S^{2n+1}$  could be extended into manifolds  $M^{2n+2}$  included into  $M^{2N+2}$  such that  $S^{2n+1} \equiv \partial M^{2n+2}$ . For instance  $P(F) = \text{STr}[e^{iF/(2\pi)}]$  would work.

A useful relationship is the Cartan homotopy formula

$$(k_{01}d + dk_{01})\mathcal{P}(F_t, A_t) = \mathcal{P}(F_1, A_1) - \mathcal{P}(F_0, A_0) \quad . \quad (7)$$

which follows by integrating

$$(l_t d + dl_t)\mathcal{P}(F_t, A_t) = \frac{\partial}{\partial t} \mathcal{P}(F_t, A_t) \quad (8)$$

over  $t$  from 0 to 1.

Then we get

$$\mathcal{L}_{2n+1} \equiv k_{01} \text{STr} (F_t^{n+1}) = (n+1) \int_0^1 dt \text{STr} (JF^n) \quad (9)$$

and

$$S = \sum_{n=0}^N \alpha_n \int_{S^{2n+1}} \mathcal{L}_{2n+1} \equiv \sum_{n=0}^N \alpha_n S_{2n+1} \quad (10)$$

or

$$S = \sum_{n=0}^N (n+1) \alpha_n \int_{S^{2n+1}} \int_0^1 dt \text{STr} (JF^n) \quad (11)$$

In ref.[1] a kinetic term was added by hand in the boundary  $\Omega^{2n}$  of the manifolds  $S^{2n+1}$  given by

$$S_K^{(2n)} = \frac{1}{2} \int_{\Omega^{2n}} d^{2n} \xi_{(2n)} \sqrt{-\gamma_{(2n)}} \left[ \gamma_{(2n)}^{ij} \text{STr} (J_i J_j) - (2n-2) \right] \quad (12)$$

or alternatively of the Born-Infeld-like form

$$\int_{\Omega^{2n}} d^{2n} \xi_{(2n)} \text{STr} \left[ \sqrt{-\text{sdet} \{ J_i J_j + (F_0)_{ij} + (F_1)_{ij} \}} \right] \quad (13)$$

or

$$\int_{\Omega^{2n}} d^{2n} \xi_{(2n)} \text{STr} \left[ \sqrt{-\text{sdet} \{ \text{STr} (J_i J_j) + (F_0)_{ij} + (F_1)_{ij} \}} \right] \quad (14)$$

where the superdeterminant is taken in the curved indices  $i, j$  of the pull-backs on  $S^d$  while the supertraces are taken on the group indices. We will ignore those kinetic terms here, even though it may be that they appear as quantum corrections to the action of eq.(3) in the quantum effective action.

From the Cartan homotopy formula for  $\mathcal{P}(F_t, A_t) = I_{2n+1}^0(F_t, A_t)$ , where the Chern-Simons (CS) form  $I_{2n+1}^0(F, A)$  is defined as

$$I_{2n+1}^0(F, A) = (n+1) \int_0^1 ds \text{STr} (A F_s^n) \quad , \quad (15)$$

where

$$A_s = sA \quad , \quad F_s = dA_s + A_s^2 = sF + s(s-1)A^2 \quad . \quad (16)$$

we get

$$k_{01} \text{STr} (F_t^{n+1}) = I_{2n+1}^0(F_1, A_1) - I_{2n+1}^0(F_0, A_0) - d \left[ k_{01} I_{2n+1}^0(F_t, A_t) \right] \quad (17)$$

Where we used that  $dI_{2n+1}^0(F, A) = \text{STr} (F^{n+1})$ . The last term is a boundary term which is given explicitly by

$$C_{2n}(F_1, A_1; A_0, F_0) \equiv -n(n+1) \int_0^1 ds \int_0^1 dt s \text{STr} (A_t J F_{st}^{n-1}) \quad (18)$$

with  $F_{st} = sF_t + s(s-1)A_t^2$  and  $A_t = tA_1 + (1-t)A_0$ .

### 3. Invariances of the Action

By construction the action of eq.(3) and the kinetic terms given above are generally covariant.

The content of this section regarding the transformation properties of CS forms and descent equations was developed in the context of anomalies in quantum field theories in refs.[2, 3, 4, 5]. I find it worthwhile to review it here because I apply it in a different context and conceptual framework.

Under (super)gauge transformations we have

$$A_r^g = g^{-1}(A_r + d)g \quad , \quad r = 0, 1 \quad (19)$$

where  $g$  is an element of the group. It follows that  $J = A_1 - A_0$  transforms covariantly if both  $A_1$  and  $A_0$  are transformed with the same  $g$

$$J^g = g^{-1}Jg \quad (20)$$

Also

$$F^g = g^{-1}Fg \quad (21)$$

Under infinitesimal gauge transformations

$$\delta_v A_r = dv + [A_r, v] \quad , \quad r = 0, 1 \quad (22)$$

Then

$$\delta_v J = [J, v] \quad . \quad (23)$$

and

$$\delta_v F = [F, v] \quad . \quad (24)$$

From the facts that  $F$  and  $J$  are gauge covariant, the cyclicity of  $\text{STr}()$  and eq.(11) it follows that the action and also the kinetic terms of eqs.(12-14) are gauge invariant.

In order to compute the change of the action under gauge transformations involving only one of the gauge fields  $A_1$  or  $A_0$  it is useful to consider elements of the gauge group  $g(x, \theta)$  function of the point  $x$  on the base manifold and a set of parameters  $\theta^\alpha$  on some 'parameter space', such that  $g(x, \theta = 0) = 1$  (the identity). In addition to the standard exterior derivative  $d = dx^\mu \frac{\partial}{\partial x^\mu}$  we define the exterior derivative in parameter space  $\zeta = d\theta^\alpha \frac{\partial}{\partial \theta^\alpha}$ . If

$$\overline{A} = g^{-1}(A + d)g = A^g \quad (25)$$

and

$$\mathcal{A} = g^{-1}(A + d + \zeta)g = g^{-1}(A + \Delta)g = \overline{A} + v \quad (26)$$

with  $\Delta = d + \zeta$  and  $v = g^{-1}\zeta g$ . We have  $d^2 = \zeta^2 = d\zeta + \zeta d = \Delta^2 = 0$ . It is easy to verify that  $\zeta \overline{A} = -D_{\overline{A}}v$  so that  $\zeta$  generates gauge transformations with

parameter  $v_\alpha = g^{-1} \frac{\partial}{\partial \theta^\alpha} g$ . The derivative  $\zeta$  corresponds to the BRS operator and  $v$  to the Fadeev-Popov ghost[2, 3, 4, 6]. Defining  $\overline{F} = d\overline{A} + \overline{A}^2$  and  $\mathcal{F} = \Delta\mathcal{A} + \mathcal{A}^2$  it is possible to check the 'Russian formula'

$$\mathcal{F}(\mathcal{A}) = \overline{F}(\overline{A}) = g^{-1} F(A) g \quad (27)$$

Considering

$$\begin{aligned} \mathcal{A}_t &= t\mathcal{A}_1 + (1-t)\mathcal{A}_0 \\ \overline{\mathcal{A}}_t &= t\overline{\mathcal{A}}_1 + (1-t)\mathcal{A}_0 \end{aligned}$$

and  $\zeta\mathcal{A}_0 = 0$ , then from the 'Russian formula' and the Cartan homotopy formula with  $\mathcal{P}_t = \text{STr}(F_t^{n+1})$  for  $\mathcal{A}_t$  and  $\overline{\mathcal{A}}_t$  we get

$$(d + \zeta)I_{2n+1}^0(\overline{\mathcal{A}}_1 + v, \mathcal{A}_0) = dI_{2n+1}^0(\overline{\mathcal{A}}_1, \mathcal{A}_0) \quad (28)$$

where  $I_{2n+1}^0(\mathcal{A}_1, \mathcal{A}_0) = k_{01} \text{STr}(F_t^{n+1}) = \mathcal{L}_{2n+1}$  correspond to the pieces of diverse dimension of our lagrangian. If we expand by the order in  $v$

$$I_{2n+1}^0(\overline{\mathcal{A}}_1 + v, \mathcal{A}_0) = \sum_{k=0}^{2n+1} I_{2n+1-k}^k(v, \overline{\mathcal{A}}_1, \mathcal{A}_0) \quad (29)$$

then we obtain the 'descent equations'

$$\zeta I_{2n+1-k}^k(v, \overline{\mathcal{A}}_1, \mathcal{A}_0) + dI_{2n-k}^{k+1}(v, \overline{\mathcal{A}}_1, \mathcal{A}_0) = 0 \quad , \quad k = 0, \dots, 2n+1 \quad (30)$$

In particular

$$\zeta I_{2n+1-k}^0(\overline{\mathcal{A}}_1, \mathcal{A}_0) + dI_{2n-k}^1(v, \overline{\mathcal{A}}_1, \mathcal{A}_0) = 0 \quad (31)$$

gives the variation of our action under a gauge transformation involving only  $\mathcal{A}_1$  (notice that  $\overline{\mathcal{A}}_1|_{\theta=0} = \mathcal{A}_1$ ) as a boundary term. A similar identity holds for gauge transformations involving only  $\mathcal{A}_0$ .

#### 4. Equations of Motion

In the case of CS gauge theory or supergravity without branes or boundaries the action is

$$S = \int_{S^{2n+1}} I_{2n+1}^0(F, A) = \int_{M^{2n+2}} \text{STr}(F^{n+1}) \quad (32)$$

then under variations of the gauge potential

$$\delta S = (n+1) \int_{M^{2n+2}} \text{STr}(D(\delta A)F^n) = (n+1) \int_{M^{2n+2}} d[\text{STr}(\delta A F^n)] \quad (33)$$

where we used  $\delta F = D(\delta A)$  and eq.(2). From Stokes theorem

$$\delta S = (n+1) \int_{S^{2n+1}} \text{STr}(\delta A F^n) \quad (34)$$

Then the equations of motion  $\frac{\partial S}{\partial A} = 0$  are [10, 11, 13]

$$\text{STr}(T^I F^n) = 0 \quad (35)$$

Whether or not these equations are related to General Relativity and/or standard Supergravity in diverse dimensions has been discussed in Ref.[9, 10, 11, 13, 17, 14].

In the case there are boundaries and branes we need to use that

$$\begin{aligned} \delta_1 J &= \delta A_1 \quad , \quad \delta_0 J = -\delta A_0 \\ \delta_r F_t &= D_t(\delta_r A_t) = d(\delta_r A_t) + [A_t, (\delta_r A_t)] \quad , \quad r = 0, 1 \\ \delta_1 A_t &= t \delta A_1 \quad , \quad \delta_0 A_t = (1-t) \delta A_0 \end{aligned}$$

Therefore we have for variations of  $A_1$

$$\delta_1 \mathcal{L}_{2n+1} = (n+1) \int_0^1 dt \text{STr}(\delta A_1 F_t^n) + n(n+1) \int_0^1 dt t \text{STr}(J D_t(\delta A_1) F_t^{n-1}) \quad (36)$$

but

$$d[\text{STr}(\delta A_1 F_t^n)] = \text{STr}(D_t J \delta A_1 F_t^{n-1}) - \text{STr}(J D_t(\delta A_1) F_t^{n-1}) \quad (37)$$

where we used  $d \text{STr}(\quad) = \text{STr}(D_t \quad)$  and the Bianchi identity  $D_t F_t = 0$ , then

$$\begin{aligned} \delta_1 \mathcal{L}_{2n+1} &= (n+1) \int_0^1 dt \text{STr}(\delta A_1 F_t^n) + n(n+1) \int_0^1 dt t \text{STr}(\delta A_1 D_t(J) F_t^{n-1}) \\ &\quad + d \left[ n(n+1) \int_0^1 dt t \text{STr}(\delta A_1 J F_t^{n-1}) \right] \quad (38) \end{aligned}$$

The last term of the second member is a boundary term. Under variations of  $A_0$  we have

$$\begin{aligned} \delta_0 \mathcal{L}_{2n+1} &= -(n+1) \int_0^1 dt \text{STr}(\delta A_0 F_t^n) + n(n+1) \int_0^1 dt (1-t) \text{STr}(\delta A_0 D_t(J) F_t^{n-1}) \\ &\quad + d \left[ n(n+1) \int_0^1 dt (1-t) \text{STr}(\delta A_0 J F_t^{n-1}) \right] \quad (39) \end{aligned}$$

If we write

$$\delta_r \mathcal{L}_{2n+1} = \text{STr}(\delta A_r Q_{2n}^{(r)}) + d \left[ \text{STr}(\delta A_r R_{2n-1}^{(r)}) \right] \quad (40)$$

where

$$Q_{2n}^{(1)} = (n+1) \int_0^1 dt F_t^n + n(n+1) \int_0^1 dt t D_t(J) F_t^{n-1}$$

$$\begin{aligned}
Q_{2n}^{(0)} &= -(n+1) \int_0^1 dt F_t^n + n(n+1) \int_0^1 dt (1-t) D_t(J) F_t^{n-1} \\
R_{2n-1}^{(1)} &= n(n+1) \int_0^1 dt t J F_t^{n-1} \\
R_{2n-1}^{(0)} &= n(n+1) \int_0^1 dt (1-t) J F_t^{n-1}
\end{aligned} \tag{41}$$

Then we can write

$$\delta_r S = \sum_{n=0}^N \alpha_n \left[ \int_{S^{2n+1}} \text{STr}(\delta A_r Q_{2n}^{(r)}) + \int_{\Omega^{2n}} \text{STr}(\delta A_r R_{2n-1}^{(r)}) \right] \tag{42}$$

or

$$\delta_r S = \int_{S^{2N+1}} d^{2N+1} x \delta_r A_m^I J^{(r)mI} \tag{43}$$

where

$$J^{(r)mI}(x^m) = \sum_{n=0}^N \alpha_n \left[ \int_{S^{2n+1}} d^{2n+1} \chi_{2n+1} \mathcal{J}_{(2n+1)}^{(r)mI} + \int_{\Omega^{2n}} d^{2n} \xi_{2n} \mathcal{J}_{(2n)}^{(r)mI} \right] \tag{44}$$

with

$$\begin{aligned}
\mathcal{J}_{(2n+1)}^{(r)mI} &= \delta^{2N+1}(X_{(2n+1)}(\chi_{2n+1}) - x^m) \text{STr} \left( T^I(Q_{2n}^{(r)})_{m_2 \dots m_{2n+1}} \right) \times \\
&\quad \times \partial_{i_1} X_{(2n+1)}^{[m} \partial_{i_2} X_{(2n+1)}^{m_2} \dots \partial_{i_{2n+1}} X_{(2n+1)}^{m_{2n+1}]} \epsilon^{i_1 \dots i_{2n+1}}
\end{aligned} \tag{45}$$

and

$$\begin{aligned}
\mathcal{J}_{(2n)}^{(r)mI} &= \delta^{2N+1}(X_{(2n+1)}^m(\xi_{2n}) - x^m) \text{STr} \left( T^I(R_{2n-1}^{(r)})_{m_2 \dots m_{2n}} \right) \times \\
&\quad \times \partial_{i_1} X_{(2n)}^{[m} \partial_{i_2} X_{(2n)}^{m_2} \dots \partial_{i_{2n}} X_{(2n)}^{m_{2n}]} \epsilon^{i_1 \dots i_{2n}}
\end{aligned} \tag{46}$$

The equations of motion  $\frac{\partial S}{\partial A_r} = 0$  are then

$$J^{(r)mI} = 0 \tag{47}$$

These equations can be interpreted as the equations found before in the case there are no boundaries or branes but now with source terms given by currents carried by the branes.

Concerning the equations of motion corresponding to extremize the action under variations of the embedding functions  $X$  it is convenient to write

$$\begin{aligned}
S &= \sum_{n=0}^N \alpha_n \left[ \int_{S^{2n+1}} d^{2n+1} \chi_{2n+1} (\omega_{2n+1})_{m_1 \dots m_{2n+1}} \partial_{i_1} X_{(2n+1)}^{[m_1} \dots \partial_{i_{2n+1}} X_{(2n+1)}^{m_{2n+1}]} \epsilon^{i_1 \dots i_{2n+1}} \right. \\
&\quad \left. + \int_{\Omega^{2n}} d^{2n} \xi_{2n} (\omega_{2n})_{m_1 \dots m_{2n}} \partial_{i_1} X_{(2n)}^{[m_1} \dots \partial_{i_{2n}} X_{(2n)}^{m_{2n}]} \epsilon^{i_1 \dots i_{2n}} \right] \tag{48}
\end{aligned}$$



where we separated the bulk and boundary contributions to  $\mathcal{L}_{2n+1}$ . In the previous expresion the dependence of  $S$  on the functions  $X$  is through the  $\omega$ 's while the dependence of  $S$  on  $\partial X$  is through the pull-back factors. The Euler-Lagrange equations for  $X_{(p)}^s$  then give

$$\left[ p \frac{\partial}{\partial X_{(p)}^r} (\omega_{(p)})_{sm_2 \dots m_p} - \frac{\partial}{\partial X_{(p)}^s} (\omega_{(p)})_{rm_2 \dots m_p} \right] \partial_{i_1} X_{(p)}^{[r} \partial_{i_2} X_{(p)}^{m_2} \dots \partial_{i_p} X_{(p)}^{m_p]} \epsilon^{i_1 \dots i_p} = 0 \quad (49)$$

In applying the Euler-Lagrange for the 'bulk'  $S^{2n+1}$  we left out a boundary term

$$\partial_{i_1} \left[ (\omega_{(p)})_{sm_2 \dots m_p} \partial_{i_2} X_{(p)}^{[m_2} \dots \partial_{i_p} X_{(p)}^{m_p]} \epsilon^{i_1 \dots i_p} \delta X_{(p)}^s \right] \quad (50)$$

We can require the boundary term to vanish, in analogy with open strings,

$$(\omega_{(p)})_{sm_2 \dots m_p} \partial_{i_2} X_{(p)}^{[m_2} \dots \partial_{i_p} X_{(p)}^{m_p]} \epsilon^{i_1 \dots i_p} \delta X_{(p)}^s = 0 \quad (51)$$

which is not to be taken as a condition on what points can be swept by the boundaries of the branes, its velocities or the allowed variations  $\delta X$  but only on the spatial derivatives of the functions  $X$  at the boundary. Alternatively we can add that term as an extra contribution to the Euler-Lagrange equations at the boundary.

If we add the kinetic terms of eq.(12-14) there would be an extra term to the current located on the boundaries  $\Omega^{2n}$  of the branes, and extra terms in the Euler-

Lagrange equations. The equations of motion of the auxiliary metrics  $\gamma$  in the kinetic terms of eq.(12) are algebraic.

## 5. Discussion

### *Connection with Superstring/M-theory*

In ref.[1] we considered a the model of eq.(3) plus the kinetic term of eq.(12) added 'by hand' for the M-theory group  $OSp(32|1)$  [24, 19, 17] and related it to IIA and IIB superstrings. That group has generators  $P_a$  (translations),  $Q_\alpha$  (generators of supersymmetries),  $M_{ab}$  (Lorentz) and  $Z_{a_1 \dots a_5}$ ,  $a = 0, \dots, 10$ . We took the symmetric trace to be the standard symmetricized supertrace in the adjoint representation of  $G^2$ .

The point we would like to make here is that that connection can be made for the action of eq.(3) alone. We consider as our candidate to the M-theory

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<sup>2</sup>For the following discussion the relevant traces of products of generators in the adjoint representation of  $OSp(32|1)$  are [1]  $\text{STr}(PP)$ ,  $\text{STr}(PQ)$  and  $\text{STr}(QQ)$ , while for the WZNW-term the relevant traces are  $\text{STr}(PPP)$ ,  $\text{STr}(PPQ)$ ,  $\text{STr}(QQP)$  and  $\text{STr}(QQQ)$ . Among these, the non-vanishing ones (after the limiting process of [1]) are normalized as

$$\text{STr}(P_a P_b) = \eta_{ab} \quad , \quad \text{STr}(P_a Q_\alpha Q_\beta) = (\gamma_a C^{-1})_{\alpha\beta}$$

action

$$S = \sum_{n=1}^5 \int_{S^{2n+1}} k_{01} \text{STr} \left( \exp \left( i \frac{F}{2\pi} \right) \right) \quad (52)$$

for  $OSp(32|1)$ . The integrals in the previous expression are supposed to pick up the differential forms of the right order. We will use the same notation as [1] and take the Inonu-Wigner limit as it was done in that paper. We need to consider a pure gauge  $A_1 = g^{-1}dg$  with  $g$  restricted to the form  $g = e^{i\hat{X}^a P_a + \theta^\alpha Q_\alpha}$ . Then essentially  $A_0 = [dX^a + i\bar{\theta}\gamma^a d\theta] + d\theta^\alpha Q_\alpha$ . We will also make  $A_0 = -A_1$ . Then proceeding as in [1] we consider a 11D slab with two 10D noncompact boundaries (each with the topology of  $R^{10}$ ) for which we identify the gauge parameters  $\hat{X}^a$  with the 10D coordinates. We will also take the pull-back of the gauge superfield  $A_1$  to be self-dual or anti-self dual  $A_1 = \pm * A_1$  with respect to some arbitrary auxiliary metric in the 2D boundaries of the 2-brane, which are contained in the 10D boundary of the 11D slab. To fix ideas we may think that metric is a 2D Minkowski metric  $\eta = (-1, 1)$ , then the (anti)self-duality condition means that we are keeping only the (left)right movers with respect to that metric. The bulk parts Chern-Simons of eq.(52) for the 2-brane give boundary WZW terms because the potentials are pure gauge, and both terms actually add because we chose  $A_1 = -A_0$ . On the other hand from eq.(18)  $C_2 = \text{STr}(A_1 A_0)$ , but the wedge product of a differential form with its dual with respect to a metric is the square with that metric. It follows that the  $C_2$  in each side of the slab would look like half the kinetic term computed with that metric (in the sense we would only have right or left movers). The 11D Majorana spinor has 32 components. We can split those 32 components in 10D either as  $(\mathbf{32}) = (\mathbf{16}_L, \mathbf{16}_R)$  or as  $(\mathbf{32}) = (\mathbf{16}_R, \mathbf{16}_L)$  (equivalently  $(\mathbf{32}) = (\mathbf{16}_L, \mathbf{16}_L)$ ), where  $_L$  and  $_R$  denote the chiralities in 10D. Choosing properly the anti-self duality or self duality conditions so that we have for instance left movers in one face and right movers in the other we can assemble left or right movers for  $X$  and for the spinors from both faces. Then each choice of the splitting of 11D spinors into two 10D spinors yield IIA or IIB superstrings. Of course the previous considerations only means that IIA and IIB strings corresponds to 'sectors of the phase space' of the theory, contributing to the quantum path integral. It would take more work to check if those configurations are actually solutions of the equations of motion.

#### Duality

We can distinguish between the base manifold coordinates  $X$  and the gauge parameters  $\hat{X}$ , and only identify them as a coordinate choice after choosing a topology for the base manifold, as done in a particular case in [1]. That means that we can get the various dualities of [25] at the level of the  $OSp(1, 32)$  algebra, corresponding to different choices of the set of operators associated to translations in 10D, by picking different  $g$ 's on the Maurer-Cartan form as above with the proper  $P_a$ , and doing the identification  $X \equiv \hat{X}$  for the corresponding  $\hat{X}$  on the appropriate 10D submanifold.

Concerning T-duality, the fact that the dimensional reduction of Chern characters are Chern characters in the lower dimension would single them out of all the possible invariant polynomials. Namely we should consider just a linear combination of terms of the form  $\text{STr}(F^{n+1})$  instead of some arbitrary combination of products of traces or some supersymmetric extension of the Euler characteristic.

#### *D-branes and K-theory*

Several recent works deal with the issue of D-branes and K-theory [30, 31]. The situation is often stated as 'K-theory is to be preferred to cohomology' or equivalently 'gauge fields are to be preferred to  $p$ -form RR-fields'. In our model, as pointed out in [1], we can mimic the RR-fields (which would then be regarded as composite) with the CS forms of one of the gauge potentials (say  $A_1$ ) which would couple (with an 'anomalous coupling') to the other one (say  $A_0$ ). The 'anomalous gauge transformation rule' of the RR field is then built in. Also K-theoretic constructions involve a doubling of the gauge fields, as our model does. It seems therefore reasonable to investigate the relationship between both approaches.

#### *Quantum theory and quantum effective action*

The quantum theory is formally defined by the path integral

$$Z = \sum_{\text{topologies}} \sum_p \int \mathcal{D}A \mathcal{D}X_{(p)} e^{iS/\hbar}$$

where it is understood that we must sum over all the gauge field configurations and brane and base space geometries and topologies. Suitable gauge fixing procedures should be used to eliminate redundancies in summing configurations corresponding to the same physical state. The topological character of the action and the quantization of the coupling constants may be taken to imply that the action of eq.(3) is already the quantum effective action. However a careful analysis of this question is clearly required.

#### *Anomalies and full gauge invariance*

It is worthwhile to notice that the variation of our action under a gauge transformation involving only one of the gauge fields has the right form to be canceled by an anomaly on the boundary  $\Omega^{2n}$  [4], as it satisfies the Wess-Zumino consistency condition and comes from a chain of 'descent equations'. It is therefore tempting to look for such a mechanism to ensure the full gauge invariance of the quantum theory under arbitrary gauge transformations of  $A_0$  or  $A_1$ <sup>3</sup>. However a priori it does not seem that we need to have anomalies at all on the even dimensional brane boundaries, as we could in principle arrange the fermion fields (the fermionic gauge fields of the supergroup) in couples of

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<sup>3</sup>That seems also desirable as making the same variation on both sides of a 't-parameter space slab' is reminiscent of 'distant parallelism', even though actually both  $A_0$  and  $A_1$  are evaluated at the same space-time point. Incidentally I wonder if this additional one dimensional t-parameter space has anything to do with F-theory [26].

opposite chirality so that the world volume theory is non-chiral. Yet if we chose to make it chiral, computing the precise form of the anomaly would be highly non-trivial, as we have *fermionic gauge fields* transforming as such under gauge transformations and reparametrizations (general coordinate transformations). It follows that the usual formulas for 'standard' (wether gauge or gravitational) or 'sigma model' anomalies would not apply. A discussion of anomalies for extended objects (which unfortunately I could not translate in any straightforward way to the problem at hand), and when and how to apply which formulas can be found in ref.[27] and [28]<sup>4</sup>. This approach seems to be the best hope to single out the gauge supergroup and the dimension of the space-time manifold<sup>5</sup>. If our model in eleven dimensions and with gauge group  $OSp(32|1)$  has as limiting cases the the five consistent superstring theories, then consistency and anomaly cancelation considerations from the later should translate into the fact that the former is the only one of our class of models that works. We can make a 'hand waving' argument in the sense that a fully consistent theory of Nature should be 'perturbatively smooth' when expanded around any 'point' of its phase space, in the vague sense that each order must be finite, even if the 'point' is not the true vacuum and the whole series does not converge.

#### *Vacuum and Phenomenology*

If the action is already the effective quantum action, as claimed, the problem of finding the vacuum reduces to finding a solution of the classical equations of motion. Doing phenomenology would require a realistic solution in the sense of having four large nearly flat 3+1 space-time dimensions (at least at some stage of cosmic evolution) and the masses and coupling constants of particle physics could be read from the coefficients of the lower order terms in a background field expansion quantization.

#### *Group manifold/Superspace formulation*

An interesting possibility would be to treat the base manifold and the fiber on the same footing by a group manifold approach. That may furthermore allow to treat the BRS operator  $\zeta$  and the exterior derivative  $d$  on a symmetric fashion, giving rise to a sort of 'double group manifold approach'. It may also be possible and useful to extend the definition of  $l_t$  and the one-dimensional t-parameter to a manifold as in [4] and treat also  $l_t$  in a more symmetrical fashion (a 'triple group manifold approach'?).

#### *Pregeometric theory*

I find very attractive the purely algebraic way in which the differential structure is treated in Refs.[2, 3, 4, 5]. It is also remarkable the contrast between the simplicity and terseness of the formalism set forth on those papers and the power and scope of those methods. I believe that is a broad hint of the kind of conceptual and mathematical framework required to describe a fundamen-

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<sup>4</sup>See also [29] and references therein. There are more recent references on brane anomalies but do not apply to the peculiarities of our model.

<sup>5</sup>It would be ironic if a model of the kind studied in [1] and the present paper is relevant to M-theory as I believe it is, as the action of the model itself is essentially an 'anomaly'.

tal theory of Nature for which the differential structure is a dynamical entity. It seems that the more fundamental formulation of such a theory must be a discrete one ('Covariant Matrix Theory') [32, 33, 34, 35, 36]. One can think that it is not possible to give physical content to the de Rham complex without giving up the smooth manifold picture of the space-time. As pointed in [35] the pregeometric approach and the geometric approach mentioned in the previous item might not be compatible, and we believe the pregeometric one is more likely to give a conceptually tight picture of physical reality. Possibly asking for a pregeometric *and* a geometric group manifold/superspace formulation at once may be like asking for Newton's Laws *and* circular planetary orbits.

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